Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

The co-ordinates of the centre of the circumscribed circle satisfy the equations,

$$\frac{\alpha}{\cos A} = \frac{\beta}{\cos B} = \frac{\gamma}{\cos C}.$$

Hence the required relation follows.

Treat, in a similar manner the equations to the escribed circles, viz: —

$$\cos \tfrac{1}{2}A \,.\, (-\alpha)^{\frac{1}{2}} + \sin \tfrac{1}{2}B \,.\, \beta^{\frac{1}{2}} + \sin \tfrac{1}{2}C \,.\, \gamma^{\frac{1}{2}} = 0, \text{ etc.}$$

[*R. H. Graves.*]

106

If m be a positive integer,

$$\sin(m-1)\varphi - x \sin m\varphi + x^m \sin\varphi$$

will contain $1 - 2x\cos\varphi + x^2$. [*A. Hall.*]

SOLUTION.

The remainder, after a division of

$$R_n = x^n \sin(m-n-1)\varphi - x^{n+1}\sin(m-n)\varphi$$

by $1 - 2x\cos\varphi + x^2$, is by actual calculation found to be

$$R_{n+1} = x^{n+1}\sin(m-n-2)\varphi - x^{n+2}\sin(m-n-1)\varphi.$$

Accordingly, the division of

$$\sin(m-1)\varphi - x\sin m\varphi,$$

by $1 - 2x\cos\varphi + x^2$, will give the successive remainders,

$$R_1, R_2, R_3, \;.\;.\;.\; R_m,$$

where $R_m = x^m \sin(-\varphi)$.

This proves the proposition. [*T. U. Taylor.*]

[Solved also by Professors Hall, Thornton, and Stone.]

◆

EXERCISES.

122

If $P_1P_2P_3$ be a triangle inscribed in an ellipse, the co-ordinates of the point of concourse of its altitudes are given by the relations,

$$2ax = (a^2+b^2)(\cos p_1 + \cos p_2 + \cos p_3) - (a^2-b^2)\cos(p_1+p_2+p_3),$$
$$2by = (a^2+b^2)(\sin p_1 + \sin p_2 + \sin p_3) - (a^2-b^2)\sin(p_1+p_2+p_3);$$

where p_1, p_2, p_3 are the eccentric anomalies of the vertices. Prove these relations, and find the values of x, y when for a given P the area of $P_1P_2P_3$ is greatest. Show that the locus of the orthocentre of all such triangles is also an ellipse.

[*W. M. Thornton.*]

123

FIND an expression for the area of a quadrilateral inscribed in an ellipse in terms of the eccentric anomalies of its vertices and the axes of the curve.

[*R. H. Graves.*]

124

THE area of the curve

$$r^2 = \frac{(a^2 - b^2)^2 \sin^2\theta \cos^2\theta}{a^2 \sin^2\theta + b^2 \cos^2\theta}$$

is one-half of that of its circumscribing circle. [*R. H. Graves.*]

125

IF M be an integer prime to N, show that the number of places in the period of the repetend M/N is a divisor of

$$\varphi(N) = N\left(1 - \frac{1}{a}\right)\left(1 - \frac{1}{b}\right)\ldots\left(1 - \frac{1}{h}\right),$$

where a, b, . . . h are the prime factors of N and $\varphi(N)$ is the number of integers less than N and prime to N. [*Wm. E Heal.*]

126

FROM the centre of each of two equal coins a coin is cut at random. If one of these random coins be placed on the other at random in a horizontal position, what is the probability that the top coin will not fall off, supposing one coin just as likely to be placed on top as the other? [*Artemas Martin.*]

127

A VESSEL of depth a, the top and bottom of which are horizontal planes, is filled with a transparent fluid, the refractive index of which at a depth z below the surface is $1 + z/a$. Two small holes being made in the top, a ray of light enters at one hole, is reflected at the bottom, and emerges at the second hole; show that the distance between the holes must not be greater than

$$2a \log_e(2 + \sqrt{3}).$$

[*William Hoover.*]

YALE SENIOR PRIZE PROBLEMS.

128

A RHUMB line which cuts the meridians at the angle of 30° is projected upon a plane tangent to the south pole. The centre of projection is: 1° at the centre of the sphere; 2° at the north pole; 3° at an infinite distance on the line of the axis. Required the equation of the projection of the curve in each case.

129

THE two equal sides of an isosceles triangle move on two fixed points. Required the equation of the locus of the centre of gravity of the triangle.

130

THERE are three concentric circles of radii a, b, and c. In the circumference of each a point is taken at random. What is the average value of the square of the area of the triangle having its vertices at these points.

131

FIND the most general equation of the curve of the fourth degree which shall consist of two equal symmetrical loops and have no other branch, thus forming approximately a figure eight. Discuss the changes which result from causing the constants to vary.

132

A BROKEN line, $ABCDE$ . . . etc., is drawn in a plane, having all its angles equal and the concavity always on the same side. Each of the successive parts BC, CD, DE, etc., is half as long as the preceding. The length and direction of AB are given and the common angle. Required the direction and distance from A of the point to which the end of the line approaches, when the construction as described is continued indefinitely.

133

A, B, C, D are four points in a plane. The inverse points (with regard to a circle in the same plane) are a, b, c, d. What relations, independent of position, exist between the sides and angles of the quadrilaterals $ABCD$ and $abcd$.

134

LET ρ_a, ρ_b, ρ_c, ρ_d, ρ_e be five vectors drawn from a common but undetermined origin to five given points A, B, C, D, E. Find five scalars a, b, c, d, e such that the vector equation

$$a\rho_a + b\rho_b + c\rho_d + d\rho_d + e\rho_e = 0$$

shall hold true for all positions of the origin.

135

A UNIFORM elastic cord weighs 100 ounces. Its length when unstretched is 100 feet. If it is laid on a smooth table and stretched, its length increases one foot for every ounce of force applied. It is thrown over two pullies twenty feet apart, and on the same level, so that the central loop balances the pendent ends. Discuss the curve and the tensions.

136

DEVELOP a method of determining solar parallax by observations of the minor planets.